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Rotor Blade Cooling in High Pressure Turbines

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A simplified model of a cooled turbine blade is used to illustrate the important features of cooling. Progress from early convection-cooled blades through to today's blades is traced with reference to the Spey and the RB 211 engines. Consideration of the internal cooling shows that where maximum cooling effectiveness is wanted the Stanton number/friction factor ratio is important. A modified Spalding and Patankar theory predicts the external heat flux in an uncooled static cascade, but empirical factors are needed when this theory is applied to engine film-cooled blades. Operational experience of the Spey HP turbine blade is discussed. A new directionally solidified multipass turbine blade has been designed and developed for the RB 211 offering performance, temperature, and life improvements. The optimization of cooling, aerodynamics, stress, and manufacture requirements plays a crucial part in the design of such a blade. Finally, several new material and process developments have recently appeared which together with cooling advances should insure continued improvements to future cooled blades.

Nomenclature

A = area of cross section or surface
c_p = specific heat
d_H = hydraulic diameter
D = diameter of film hole
f = friction factor
h = convective heat transfer coefficient
l = length
L = rib height
N_c = cooling modulus
N_F = film blowing rate
N_{St} = Stanton number
N_w = coolant flow ratio
p' = total pressure
s = rib spacing
S = surface area/unit length
T = absolute temperature
u = velocity
w = mass flow rate
w^+ = nondimensional mass flow rate
ϵ = cooling effectiveness
η_c = cooling efficiency
η_F = cooling efficiency of blade with film cooling
ρ = density

Subscripts

c = coolant
ce = coolant exit
ci = coolant inlet
cs = coolant for standard blade
CD = compressor delivery
F = film
g = gas
m = metal
R = rough
S = smooth

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I. Introduction

SINCE cooled blades first entered commercial airline service with the Conway engine in 1960, steady improvements in cooling have raised turbine entry temperature from below 1000°C to today's level of over 1350°C. Early blades relied upon convection cooling but present-day blades are cooled by mixed systems employing both convection and film cooling.

The introduction of cooling into an otherwise uncooled blade brings with it certain disadvantages, including more difficult manufacture, loss of turbine aerodynamic performance, loss of engine cycle efficiency, and increased stress levels in the blade.

These difficulties can combine so that the first 50°C worth of metal cooling are absorbed in getting back to the engine thrust and blade life situation that existed with the uncooled blade. It is therefore vital that cooling designs be properly optimized with respect to the actual engine situation.

This paper describes some of the considerations that now come into the design of the cooling system for the high pressure (HP) turbine blades of a modern commercial fan engine.

II. Convection Cooling Model

The analysis of cooled blades requires the solution of the equations governing heat flow through the solid blade given the internal and external boundary distributions of convective heat transfer coefficients. To solve this problem, Rolls-Royce has developed a linked suite of interactive computer programs known as TACITUS, covering the aerodynamic and heat transfer routines needed for the design of cooled turbine blades.

Although such analytical solutions are nowadays needed for detailed blade design, the principles of blade cooling are best highlighted by a much simpler approach. Halls[1] introduced the idea of a "standard blade" having an infinite thermal conductivity and wherein the cooling air warmed up to the uniform blade temperature before leaving the blade.

Equating the heat entering the blade to the cooling air enthalpy rise for the standard blade gives:

$$\int_o^A h_g \left(T_g - T_m\right) dA = w_{cs} c_{p,c} \left(T_m - T_{ci}\right) \tag{1}$$

also equating the enthalpy rise for the standard and actual blades.

$$w_{cs} c_{p,c} \left(T_m - T_{ci}\right) = w_c c_{p,c} \left(T_{ce} - T_{ci}\right) \quad (2)$$

which leads to the concept of cooling efficiency defined as:

$$\eta_c = w_{cs} / w_c \quad (3)$$

thus from Eq. (2)

$$\eta_c = \frac{T_{ce} - T_{ci}}{T_m - T_{ci}} \quad (4)$$

Equations (1), (2), and (4) lead to,

$$\epsilon_m = \frac{w_c^+ \eta_c}{1 + w_c^+ \eta_c} \quad (5)$$

where

$$\epsilon_m = \frac{\bar{T}_g - T_m}{\bar{T}_g - T_{ci}} \quad (6)$$

and

$$w_c^+ = \frac{w_c c_{p,c}}{h_g S_g l_g} \quad (7)$$

Equation (5) shows that high effectiveness requires both high cooling flow and efficiency in a convectively cooled blade.

Coolant Flow Ratio

The turbine engineer is equally concerned with the coolant flow ratio N_w since this is the quantity which determines loss of turbine aerodynamic or cycle efficiency.

$$N_w = w_c / w_g \quad (8)$$

Now $w_c{}^+$ and N_W are related by:

$$w_c^+ = \frac{N_w c_{p,c} A_g}{N_{St,g} c_{p,g} S_g l_g} \approx \frac{N_w \%}{2.5} \quad (9)$$

Cooling Efficiency

It can be shown that,

$$\eta_c = 1 - \exp(-N_c) \quad (10)$$

where

$$N_c = N_{St,c} \cdot S_c l_c / A_c$$

and therefore

$$N_c = N_{St,c} \cdot 4 \cdot lc / d_H \quad (11)$$

Efficient cooling therefore requires either high Stanton numbers or large length diameter ratio cooling passages.

Comparison of Convection-Cooled Blades

The results of the previous analysis are summarized in Fig. 1.

Turbine blades used in earlier Rolls-Royce engines were purely convection cooled, fed by low pressure preswirled cooling air which flowed up numerous radial holes in the blade and was ejected at the tip. The air supply pressure was too low to permit film cooling of the hot leading edge and

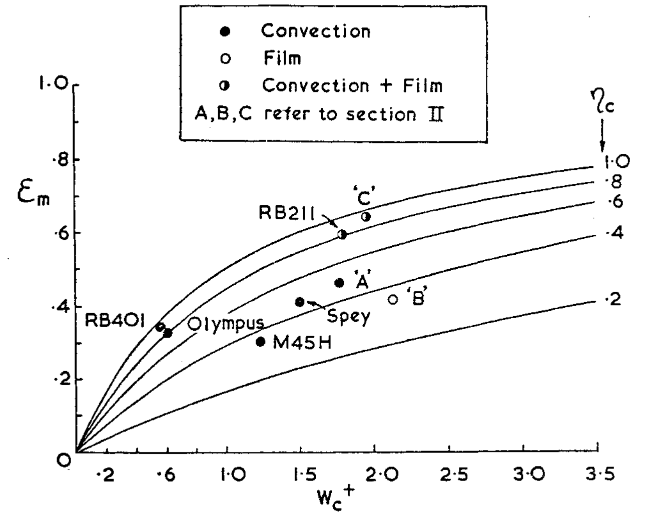

Fig. 1 Cooling performance of some Rolls-Royce HP turbine blades.

pressure surface regions. The cooling characteristics of several of these early blades are shown in Fig. 1; the blades employed sufficiently large l_c / d_H ratios to achieve the values of cooling efficiency needed. The temperature distribution in an early convection cooled RB 211 development blade is shown in Fig. 2. The cooling is uneven varying from over 500°C in the core to less than 70°C at the extremities.

Film-Cooled Blades

Nowadays most HP turbine blades employ a large amount of film cooling to cope with the more critical areas. The simple convection-cooled model can be extended to cope with film cooling with the following results:

$$\epsilon_m = \frac{w_c^+ \eta_c + \bar{\epsilon}_F (1 - \eta_c)}{1 + (w_c^+ - \bar{\epsilon}_F) \eta_c} \quad (12)$$

where

$$\epsilon_F = \frac{\bar{T}_g - \bar{T}_F}{\bar{T}_g - \bar{T}_{ce}} \quad (13)$$

also

$$\eta_F = \frac{1}{w_c^+} \left[\frac{\bar{\epsilon}_F + (w_c^+ - \bar{\epsilon}_F) \eta_c}{1 - \bar{\epsilon}_F} \right] \quad (14)$$

The values of ϵ_m and η_F from Eqs. (12) and (14) still satisfy Eq. (5).

A predominantly film-cooled blade using an RB 211 external profile is also included in Fig. 2 and shows a much flatter distribution with over 200°C of cooling at the cost of a reduced cooling efficiency compared to the convection blade. The nondimensional cooling parameters of the two blades shown in Fig. 2 are compared in Fig. 1.

Combined Convection- and Film-Cooled Blades

To explore the possibilities of combined film and convection cooling we can imagine a hypothetical engine blade which combines the good levels of film and convection cooling of blades A and B indicated in Fig. 1 by means of Eqs. (12) and (14). The results are shown in Fig. 1 as blade C and compared with the measured cooling achieved on a modern RB 211 multipass blade.

The RB 211 multipass blade manufactured in directionally solidified cast superalloy has been developed for the RB 211-22B and –535 engines. It entered service in late 1979 and Fig.

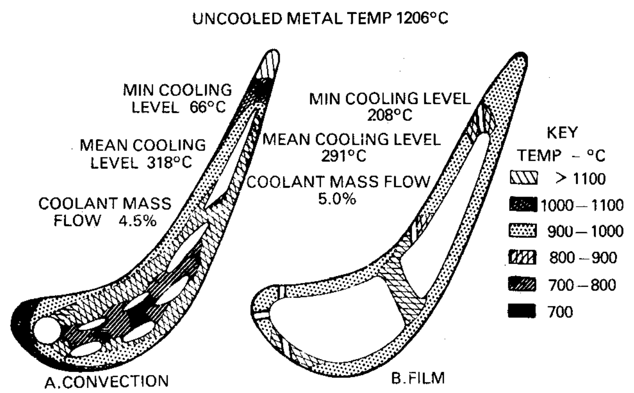

Fig. 2 Characteristics of convection- and film-cooled blades.

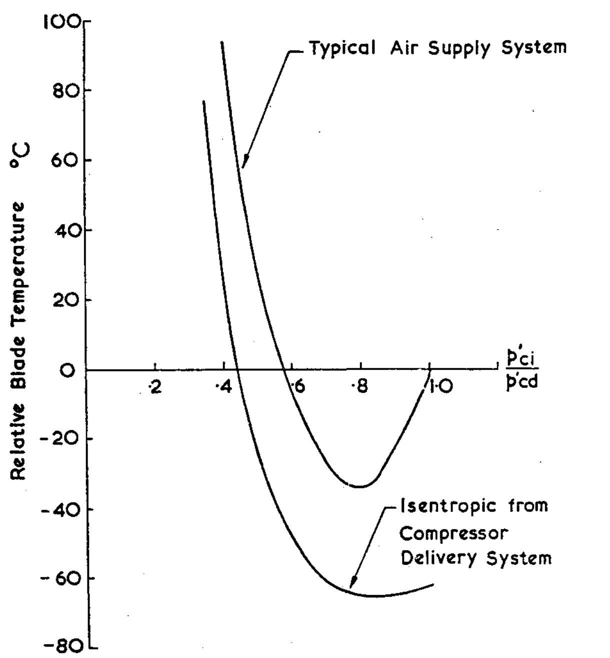

Fig. 3 Variation of turbine blade mean temperature with cooling air pressure.

1 shows it compares well with the hypothetical blade "C." Its design is considered in more detail in Sec. VI.

Cooling Air Supply

Cooling air for HP blades is generally taken from the last HP compressor stage and then expanded through an air supply system down to the blade inlet conditions. Actual engine systems suffer from pressure losses and heat pickup so that isentropic efficiencies for this process are generally low, representing a loss to the engine cycle and producing blade cooling air supply temperatures around 100°C hotter than isentropic. With blade cooling effectiveness over 0.5, this is equivalent to over 100°C on the turbine entry temperature as far as the blade is concerned. Improved air supply systems will permit significant increases in turbine entry temperature or savings in cooling airflow and would also have the added advantage of reducing the temperature of the turbine disk.

Optimum Inlet Pressures

The dependence of cooling air inlet temperature upon the inlet pressure leads to an optimum inlet pressure which minimizes blade temperature for a convection-cooled blade. Figure 3 shows that the temperature of a typical convection-cooled blade is lowest when the air supply pressure is around 80% of the engine HP compressor delivery pressure. Operating above this pressure wastes cooling air.

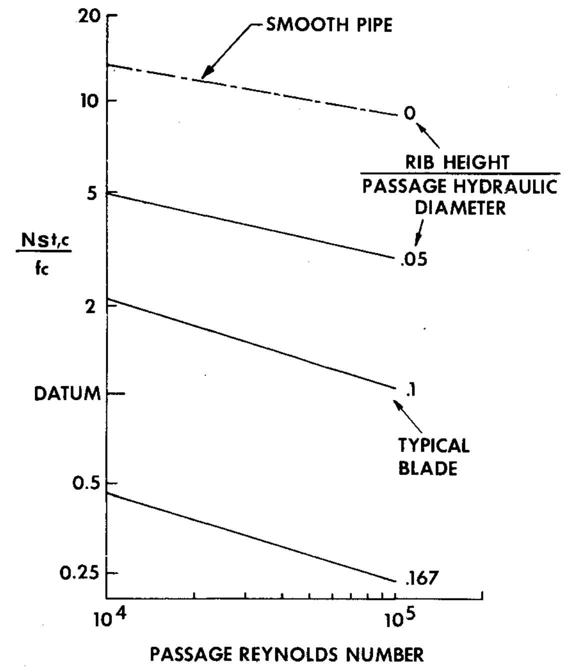

Fig. 4 Variation of $N_{St,c}/f_c$ with Reynolds Number and rib height for rectangular cooling passages with transverse ribs on two opposite walls.

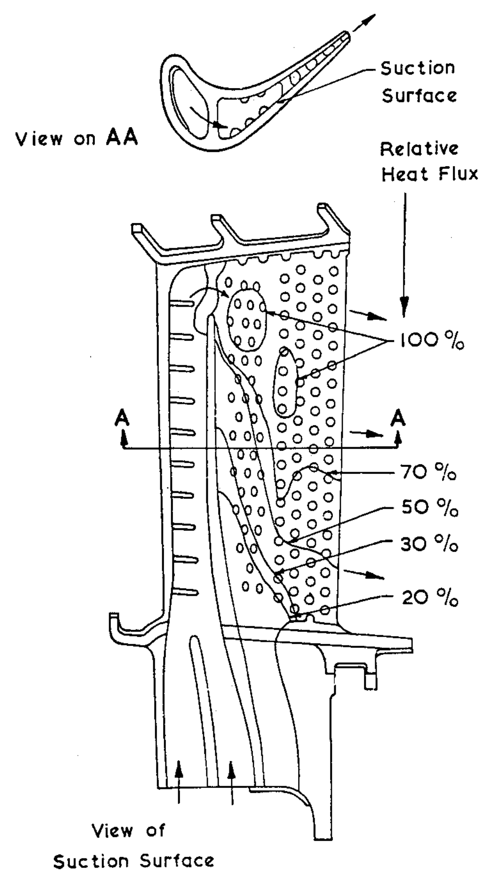

Fig. 5 Measured distribution of blade internal heat flux at engine Reynolds and Mach numbers.

III. Internal Heat Transfer

Heat is extracted from the metal walls inside the blade by forced convection increased locally where extra cooling is needed. The methods commonly adopted include transverse or longitudinal ribs, pimples, pin fins, or the impingement of

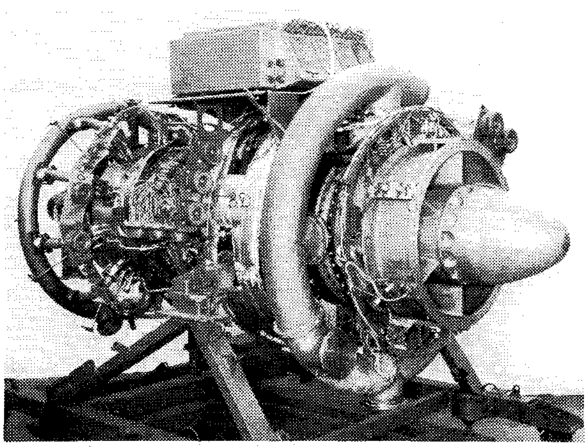

Fig. 6 Advanced demonstrator HP spool.

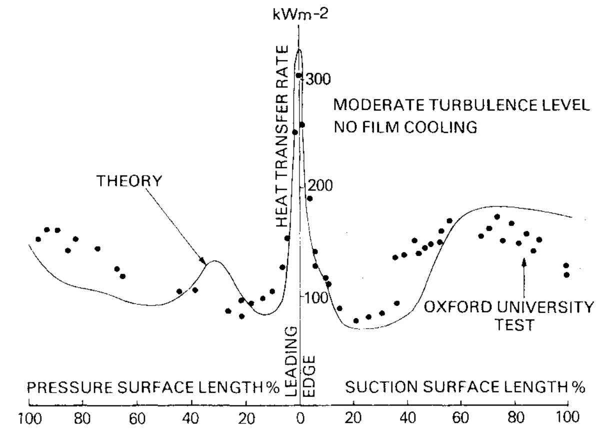

Fig. 7 Heat transfer rates around RB 211 HP turbine blade.

small jets of cooling air onto the surface. Design data are obtained in suitable test rigs which measure the heat transfer and pressure drop characteristics for the various cooling methods under static conditions, e.g., see Fig. 4. Such data may not apply to some turbine blades where the internal flows are complicated by the nonuniformity of the passages and the effects of blade camber and rotation.

It is becoming increasingly important to measure internal heat transfer rates on models which reproduce the internal conditions. Figure 5 shows the wall heat flux distribution inside the suction surface of a small HP blade. The regions of locally high heat transfer toward the tip are due to air impacting on the concave suction surface wall. The low heat transfer region toward the root corresponds to a pocket of slowly recirculating flow. Measurements such as these can now be obtained very early in the development of an engine blade and any problems identified are corrected.

Rotating effects are examined by testing blades in an actual engine HP spool (Fig. 6) where all the internal and external conditions are reproduced. Such testing is expensive, but if carried out early enough can save a lot of money by identifying cooling problems before they affect the engine development program.

Minimizing Cooling Flow

Minimizing the cooling flow means designing the blade internal cooling system for the highest cooling efficiency. Equations (10) and (11) show that the passage length/diameter ratio must be selected to suit the Stanton number. Artificial roughening gives high Stanton numbers and permits a lower passage length/diameter ratio, which offers manufacturing advantages in terms of stiffer ceramic cores for the casting process and fewer passages in the blade. Multipassing helps in the same way by producing a large passage length/diameter ratio coupled with a large passage size. These methods can also reduce blade weight and stresses in the turbine disk.

Maximizing the Effectiveness

Maximizing the effectiveness or minimizing the blade temperature for a given cooling air feed pressure and temperature means selecting a cooling method with the best combination of heat transfer and pressure drop characteristics. For the bulk of the blade, the combination to be maximized is $N_{St,c} \cdot f_c^{\,-1.0}$, however for a small critical area such as the leading edge it becomes $N_{St,c} \cdot f_c^{\,-0.5}$.

Generally, for any artificially roughened surface of the scale used for turbine blades increases f_c faster than $N_{St,c}$, see Fig. 4. We should therefore get higher effectiveness with many small smooth passages than with fewer large roughened ones. Where the roughness height is less than 5% of the passage diameter, roughening can yield higher values of $N_{St,c} \cdot f_c^{\,-0.5}$ and so locally improve small regions at the expense of the rest of the blade.

IV. External Heat Transfer and Film Cooling

External heat transfer coefficients are derived from a finite difference boundary layer calculation based upon the procedure of Ref. 2. The input consists of the velocity distribution around the blade section, details of the geometry, the surface temperature distribution, and the freestream turbulence intensity. The program integrates the momentum and enthalpy equations around the profile and predicts the boundary layer development and heat transfer coefficient distribution. The equations solved are formulated using an effective viscosity model based upon the Prandtl mixing length hypothesis. Transition between laminar and turbulent flow, the effect of surface curvature and freestream turbulence are treated semi-empirically by a similar procedure to that described by Forest.[3]

Figure 7 compares the theoretical predictions with the results of cascade tests for an RB 211 aerofoil without film cooling. The agreement is generally good, particularly around the leading edge and suction surface. These cascade tests were run over a range of Reynolds numbers and turbulence levels in the Oxford University Engineering Laboratory Transient Cascade Facility.[4] In the engine in the presence of films the pressure surface can exhibit very high levels of heat transfer. For this reason external heat transfer prediction has to be supported by data derived from measured blade temperatures. In this way empirical factors have been created to bridge the gap between cascade and engine test. Work is continuing to model this theoretically.

Film Cooling

The behavior of the film varies with its initial momentum, angle of ejection, film hole shape, hole pitching, mainstream gas Mach number, blade curvature, and pressure gradients; consequently theoretical treatments are not generally reliable and it is normal to test different geometries experimentally. At Rolls-Royce, adiabatic wall film effectiveness values are measured on a large-scale insulated aerofoil whose surface is fitted with thin film temperature gages downstream of the film holes. The results obtained do not include the effect of the film on the convective coefficient distribution, which would be substantial close to the injection region. This effect has so far been investigated mainly by reference to engine and HP spool test results.

Figure 8 illustrates typical adiabatic wall effectiveness results measured on the large-scale rig and plotted against the conventional blowing rate N_F

$$N_F = (\rho u)_F / (\rho u)_g \tag{15}$$

It can be seen that film effectiveness is very dependent upon N_F. If the blowing rate is too high, the film separates from the blade surface when the conventional round film holes are

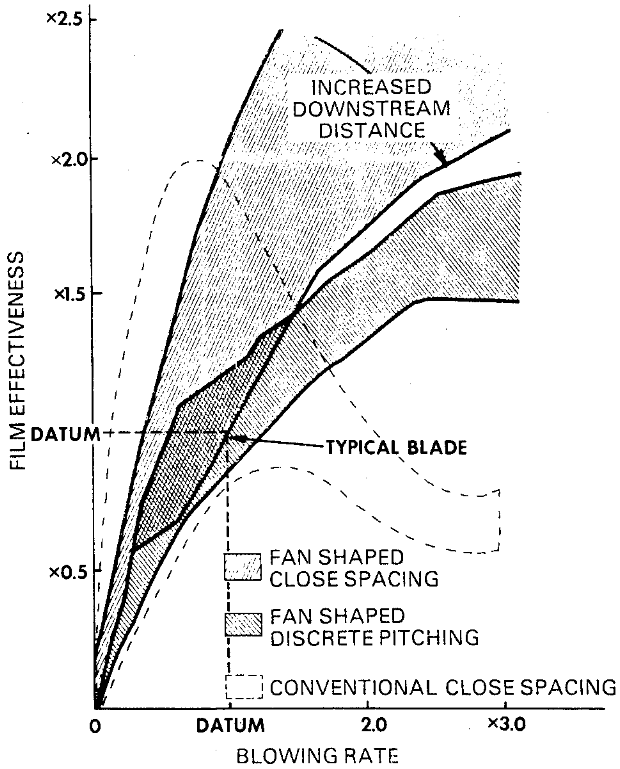

Fig. 8 Film cooling performance of fan-shaped and circular holes.

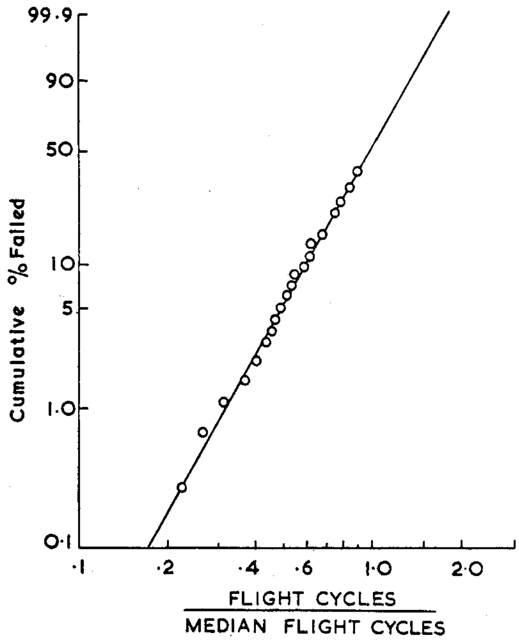

Fig. 9 Variation of HP turbine blade life in airline service for a modern civil engine.

used. Figure 8 shows that fan shaped holes, where the film air diffuses before ejection, delay the separation and provide higher film effectiveness levels at high blowing rates. Blowing rates tend to be largest where the external mainstream Mach numbers are lowest. They generally reach a maximum of 2.0 close to the leading edge falling to 1.0 along the suction surface.

In the design of modern turbine blades the blade cooling engineer must pay attention to film cooling pressure margins, particularly in multipass systems. In this type of system it may be possible to have one film flowing and another one ingesting, creating a local hot spot. To validate theoretical calculations engine tests are carried out to determine the onset of ingestion for critical films, and to provide sensible margins to cover manufacturing tolerances.

V. Blade Life

When designing a blade it is necessary to balance the conflicting requirements of low initial cost, good aerodynamic performance, and long life. Blade life is subject to wide variation in a fleet of aircraft due to the combined effects of variability in service operation plus engine-to-engine and blade-to-blade scatter. The range of blade life found is typically three to one, it is therefore important and must be allowed for at the design stage. The Weibull distribution[5] fits the life variations found in service. Figure 9 illustrates the variation of life as a Weibull line for an RB 211 HP turbine blade, based upon data given in Ref. 6.

Service Experience

The Spey HP1 turbine blade has been in commercial service since 1966. This forged blade is purely convection cooled and is shrouded carrying a single tip fin. Experience has been good and a typical life in service operation is some 4860 h. This life is defined statistically using Weibull with a 10% chance of an engine set experiencing a blade failure at 4860 h, based upon a typical 2 h flight cycle experienced in a Trident or BAC 111 operation. To insure a high standard of reliability, the service operation is planned such that blades are replaced before failure occurs. This achieves a balance between reliability and operating economics. In practice, 50% of the turbine blades are replaced before the 10% failure life, this policy coupled with a rigorous inspection of turbine blades at fixed intervals ensures minimum unplanned removals.

The Spey blade is shown in Fig. 10 with the new RB 211 turbine blade. The Spey is a low bypass engine and the takeoff turbine entry temperature is significantly higher than at climb and cruise conditions. Details of a typical Spey flight cycle are given in Table 1, which shows the turbine temperature for the bulk of the flight to be at least 115°C lower than at takeoff. As a result the critical flight condition for a Spey blade is at takeoff and over 75% of its total life is used up by airfoil thermal stress during this process.

The Spey has a good reliability record in service. The failure, which is generally in a thermal fatigue mode, occurs at a predictable time, within a predictably narrow scatter band. In addition, the use of forged material and a tip shroud

Table 1 Engine temperature cycle

Item	Spey		RB 211	
	Rating, °C	Time	Rating, °C	Time
Temperature increase at takeoff from idle	+570	5 secs	+720	8 secs
Takeoff temperature	1152	1½ min	1232	2 min
Reduction at climb	115	15 min	30	21 min
Reduction at cruise	145	50 min	185	60 min
Flight idle and cruise	...	27 min	...	22 min
Ground idle and taxi	...	25 min	...	25 min
Flight duration		2 h		2 h, 10 min

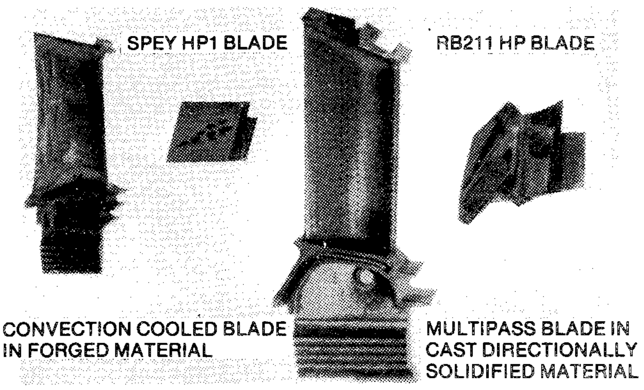

Fig. 10 Comparison of Spey and RB 211 turbine blades.

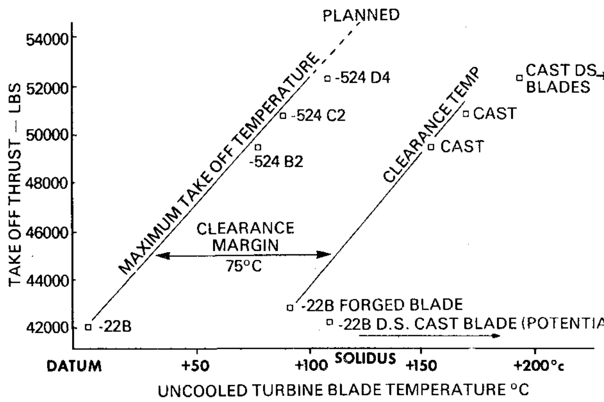

Fig. 11 Temperature vs thrust for RB 211 derivatives.

insures very little secondary damage in the case of a blade airfoil failure. This is an extremely important feature of a shrouded turbine blade.

On modern bypass engines, such as the RB 211, the blade cooling situation is far more strenuous. Here, not only is the temperature increase at takeoff greater, but the reduction during climb is only approximately 30°C. This results in an increase in creep damage compared to the Spey.

VI. RB 211 Multipass Blade

During the late 1960's and early 1970's turbine entry temperature (TET) increased rapidly. This trend still continues today, but conscious efforts are now made to minimize the TET at a thrust to improve engine life. The reason for high TET is quite clear if one considers the thrust/temperature relationship. Historically it has been possible to increase the thrust of the RB 211 engine from 42,000 to 53,000 lb by TET increase with modest core change (see Fig. 11).

Reliability is a prime factor in all engine applications so engines are operated on development test beds at much higher temperatures than would normally be experienced in service. This insures that potential service problems show up during development testing. On the RB 211 this clearance margin is at least 75°C relative to normal hot day takeoff temperature and allows for transient overshoot runway ram rise and transient deterioration.

The HP turbine blade design in the RB 211 has evolved from the earliest simple convection-cooled forged blades to the new multipass blade which has been cleared at 42,000 lb thrust rating and offers considerable future stretch in other RB 211 variants.

Design Concepts

The multipass concept had already been demonstrated on earlier experimental RB 211 variants and also undergone a rigorous test program on an advanced core demonstrator. The

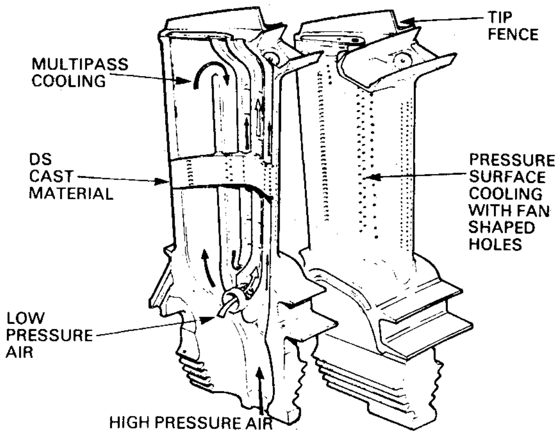

Fig. 12 RB 211 multipass HP turbine blade.

design employs unidirectionally solidified material offering improved creep strength and resistance to thermal fatigue.

The need for high cooling effectiveness at a minimum cooling flow dictates a cooling system with a good cooling efficiency, see Eq. (5). This was achieved here by a combined system of multipass, transverse ribs, and extensive film cooling.

To assist manufacture, a single-piece ceramic core was adopted for the multipass blade. Here the emphasis has been on ruggedness to obtain a high casting yield and avoid core movement problems during the lengthy directional solidification process. The design shown in Fig. 12 comprises five spanwise cooling passages, three of which are linked to form a triple pass system, the other two remaining as single pass.

Performance Features

To achieve maximum aerodynamic efficiency the blade has several features which are unique to Rolls-Royce. These include a split feed system consisting of high and low pressure cooling air supply, together with a tip deflector cap. On any cooling air feed system there is inevitably some leakage past the high pressure seals. If this flow is allowed to leak into the mainstream turbine annulus, it will cause serious performance penalties. The advantage of a split HP/LP feed system is that it makes use of this leakage air to cool the central core of the blade. The higher quality high pressure air is then used to cool the extremities of the blade. To minimize the performance penalties of cooling airflow on the engine cycle, the cooling air that is pumped up the turbine blade is ejected from a tip fence in the direction of the mainstream gas flow. This enables work to be extracted from the coolant flow worth 0.75% SFC.

To reconcile the demands of cooling and aerodynamics it is necessary to optimize the ejection of film cooling air on the blade. Extensive experimental investigations over a period of eight years have quantified film performance both in terms of cooling effectiveness and aerodynamic loss, see Figs. 8 and 13. The multipass blade employs film cooling extensively on the pressure surface where it is less penalizing and fan-shaped holes where the blowing rate is high.

The complex design interaction between the needs of manufacture, cooling, performance, and stress has led in Rolls-Royce to the setting up of a team of specialists covering these fields and integrated into a single group engaged in turbine design.

Blade Temperature Prediction

Detailed blade temperature predictions were originally made at the design stage using TACITUS. These were later confirmed by cascade rig and engine test results. Special engine tests were carried out using thermocouples and tem-

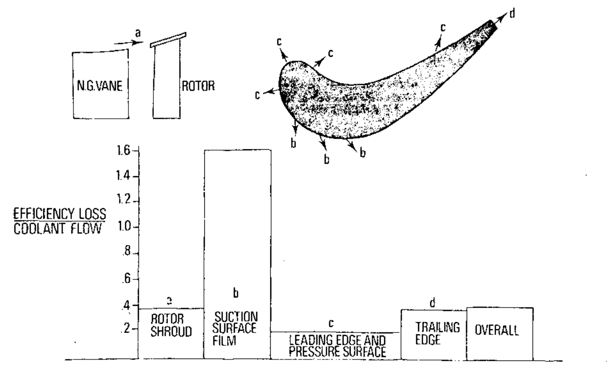

Fig. 13 Breakdown of turbine efficiency loss associated with rotor cooling flows.

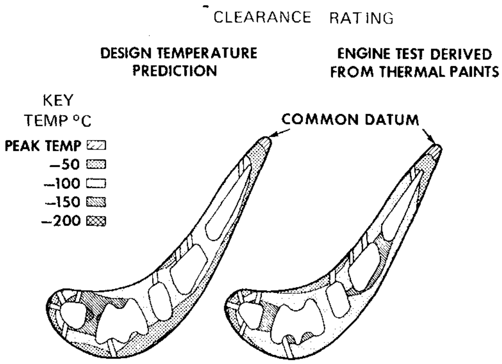

Fig. 14 Theory and test for multipass RB 211 blade.

perature sensitive paints to determine the surface temperature distribution of the blade in the actual engine environment where it is subject to rotation and combustor temperature profile effects. TACITUS was used again after the engine tests, this time to construct the blade internal temperature distribution corresponding to the measured surface temperature. Temperature contours for the multipass blade are shown at midspan in Fig. 14 which compares the design predictions with the analyzed engine data.

It is clear that the agreement is excellent on the pressure surface and at the extremities of the blade. The ability to handle large quantities of data using the TACITUS system had led to significant improvements in blade cooling design over the last five years. It will be used in the future to improve the temperature prediction on the suction surfaces of the turbine blades (see Fig. 14).

VII. Future Developments

The need to increase the energy conversion efficiency of engines will lead to further increases in engine pressure ratio and temperatures. At the same time we must improve the reliability and life of turbine blades in order to reduce costs.

Improved Materials and Manufacture

There has recently been increased interest in the processing and manufacture methods of superalloy blades caused by difficulties in further improving basic alloy compositions. The introduction of unidirectional solidification has brought improvements, notably in thermal fatigue life. It is hoped that the future use of single crystal blades, rapid solidification rate, powder metallurgy, directionally solidified eutectics, and oxide dispersion strengthened alloys will permit further increases in engine temperatures. A detailed discussion of the impact of recent material advances on cooled turbines is contained in Refs. 7-9.

Cooling Improvements

To increase aerodynamic performance and reduce small hole blockage problems for engines operating in desert areas we need to reduce our present dependence upon film cooling by maximizing the internal convection.

It is now possible to produce long smooth bore holes of very small diameter in the large numbers required to cool a turbine blade so that very efficient cooling can be achieved without the excessive pressure losses produced by artificial roughening and multipassing. Recent advances in the United States have led to the introduction of wafer blades[8]. There are, however, limits to convection alone; with blades now approaching cooling efficiencies of unity, the law of diminishing returns applies to further internal complexity. In addition, the low thermal conductivity of the superalloys leads to large temperature gradients in the high heat flow regions of convectively cooled blades.

Possible solutions to these problems include the introduction of thermal barrier coatings to insulate the blade,[9] liquid metal cooling, and reducing the temperature of the cooling air. Cooler cooling air becomes attractive now that blade cooling effectiveness values exceed 0.5. There appear therefore to be several alternative avenues open, each offering the possibility of substantially improving the cooling of HP turbine blades in the foreseeable future. Which ones will stand the test of time remains to be seen.

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